paper - XIII
classical mechanics. ॐ नमः शिवाय

①

Constraints and its classification :

Constraints :- Restrictions of geometric or Kinematical nature imposed on the positions and velocities of the system of particles are called Constraints.

A System with such Constraints is called Constrained System.

A System without any Constraint is called free System.

Analytically, a Constraint is expressed by the relation

$$f(t, r_\nu, \dot{r}_\nu) = 0 \quad \text{——— } (1)$$

$$(\nu = 1, 2, 3, ---, N)$$

where t is the time, r_ν and $\dot{r}_\nu$ are the position vector and velocity of the νth particle respectively.

If in equation (1), $\dot{r}_\nu$ is absent then the Constraint $f(t, r_\nu) = 0$ is called finite or geometric Constraint.

If in the finite Constraint t does not involve explicitly i.e. $\frac{\partial f}{\partial t} = 0$, then the finite Constraint is called stationary.

If in the finite Constraint t involves explicitly i.e. $\frac{\partial f}{\partial t} \neq 0$, then the finite Constraint is called non-stationary.

Generally the Constraint $f(t, r_\nu, \dot{r}_\nu) = 0$ is called differential or Kinematical Constraint.

It may or may not be integrable.

Thus we have four fold classification of Constraints —

(i.) geometric - Stationary and non-Stationary

(ii.) Differential - Integrable and non- integrable.

Potential type of forces - If there exists a force function or a potential function of position vector r_j such that the force may be derived by differentiating force function with respect to r_j, then the force is called potential type of force.

If there does not exist any force function or potential function of position vectors r_j then the force is called non potential type of force

Classification of dynamical system :

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There are eight fold classifications of dynamical system (System of particle in motion).

(i.) Holonomic and non- holonomic

(ii.) Scleronomic and Rheonomic

(iii.) Conservative and non- Conservative

(iv.) Simple and non- Simple.

Holonomic dynamical System :

Any free dynamical System or a dynamical System with integrable differential Constraints is Called holonomic System.

Non-holonomic System : A System with non integrable differential Constraint is Called non-holonomic System.

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Scleronomic System : A System with finite stationary Constraint is Called scleronomic System.

Rheonomic System : A System with finite non-Stationary Constraint is Called rheonomic System.

Conservative System : A System with potential type of forces is Called Conservative System.

Non-Conservative System : A System with non potential type of forces is Called non Conservative System.

Simple System : A System with holonomic or Scleronomic or Conservative or holonomic and Scleronomic or holonomic and Conservative or scleronomic and Conservative or holonomic and scleronomic and Conservative is Called Simple System.

Non-Simple System : A System with non-holonomic or rheonomic or non-Conservative or non-holonomic and rheonomic or non-holonomic and non-Coservative or rheonomic and non-Conservative or non-holonomic and rheonomic and non-Conservative is Called Non-Simple System. As it is very difficult to deal with such system

Illustration and example.

(i) A particle is Constrained to move over a surface whose equation may be written as $f(\vec{r}) = 0$. Then the equation of Constraint may be written as $f(x, y, z) = 0$

This Constraint is finite and stationary.

If the surface moves, then the time t enters in the equation of the surface explicitly. Then the equation takes the form $f(t, x, y, z) = 0$

This is a finite and non-stationary Constraint. In this Case the dynamical system is holonomic and rheonomic.

(ii) Two particles are Connected by a rod of Constant length l. Then the Constraint equation may be written as

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 - l^2 = 0$$

This is a finite stationary Constraint.

The dynamical system is holonomic scleronomic

(iii) Two particles on a plane are connected by a rod of constant length l and they are constrained to move in such a way that the velocity of the middle point of the rod is in the fixed direction inclined at an angle α with the x-axis. Then the equation of constraints are given by

$$z_1 = z_2 = 0$$

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 = l^2$$

$$\frac{\dot{x}_1 + \dot{x}_2}{\cos\alpha} = \frac{\dot{y}_1 + \dot{y}_2}{\sin\alpha}$$

Here the constraint is integrable differential and the dynamical system is holonomic.

(iv) Two particles in a plane are connected by a rod of constant length l and they are constrained to move in such a way that the velocity of the middle point of the rod is along the rod. Then the equation of the constraint may be written as

$$z_1 = z_2 = 0$$

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 = l^2$$

$$\frac{\dot{x}_1 + \dot{x}_2}{x_1 - x_2} = \frac{\dot{y}_1 + \dot{y}_2}{y_1 - y_2}$$

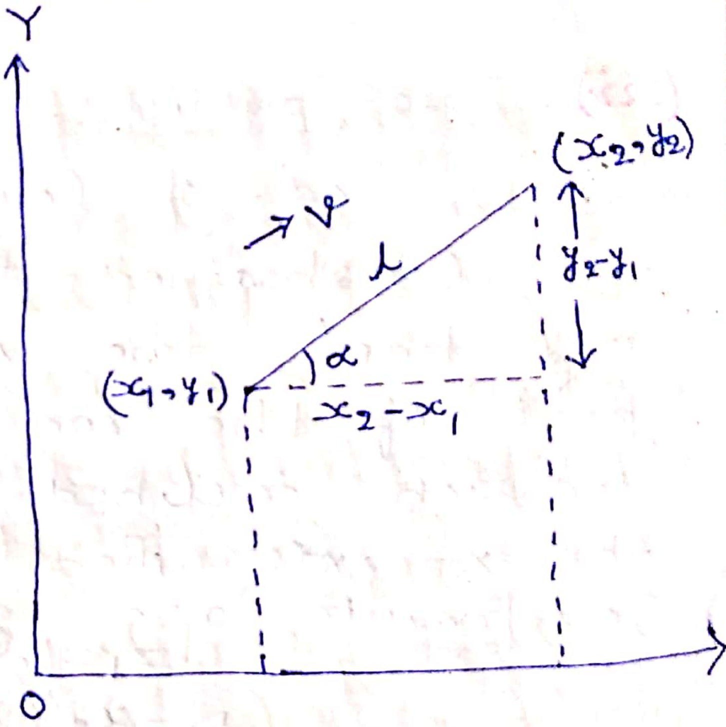
Here the constraint are non-integrable differential and the system is non-holonomic.

(4)

$$v\cos\alpha = \dot{x}_1 + \dot{x}_2$$

$$v\sin\alpha = \dot{y}_1 + \dot{y}_2$$

$$\frac{\cos\alpha}{\sin\alpha} = \frac{\dot{x}_1 + \dot{x}_2}{\dot{y}_1 + \dot{y}_2}$$



$$\cos\alpha = \frac{x_2 - x_1}{l}$$

$$\sin\alpha = \frac{y_2 - y_1}{l}$$

$$\therefore \frac{\cos\alpha}{\sin\alpha} = \frac{x_2 - x_1}{y_2 - y_1}$$

$$\therefore \frac{\dot{x}_1 + \dot{x}_2}{x_1 - x_2} = \frac{\dot{y}_1 + \dot{y}_2}{y_1 - y_2}$$